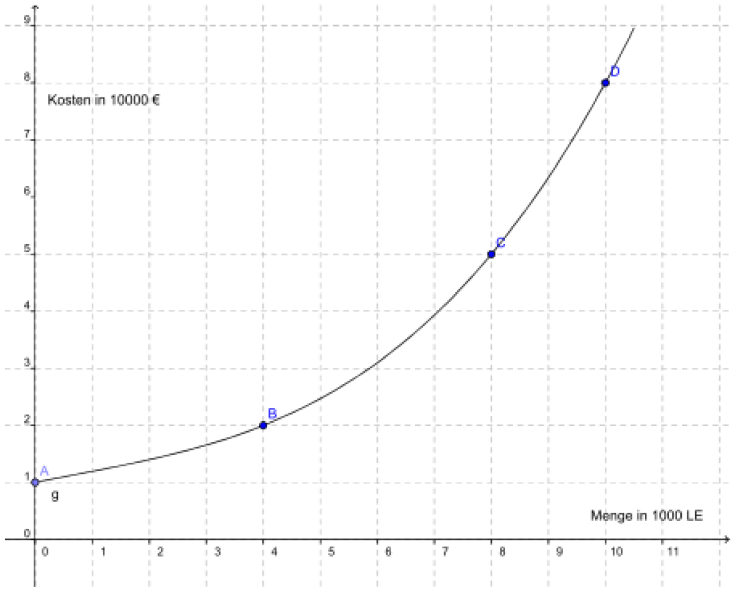
Progressive Kosten

1 Aufgabe

Figure 1:



2 Lösung

2.1 Eingabe

```
--> x1:0;K1:10000;
(%o1) 0
(%o2) 10000
```

```
--> x2:4000;K2:20000;
(%o3) 4000
(%o4) 20000
```

```
--> x3:8000;K3:50000;
(%o5) 8000
(%o6) 50000
```

```
--> x4:10000;K4:80000;
(%o7) 10000
(%o8) 80000
```

2.2 Verarbeitung

Ansatz Kostenfunktion dritten Grades

```
--> g(x,K):=K=a*x**3+b*x**2+c*x+d;
```

(%o9) $g(x,K):=K=a\,x^3+b\,x^2+c\,x+d$

Gleichungssystem

```
(%i9) g1:g(x1,K1);
      g2:g(x2,K2);
      g3:g(x3,K3);
      g4:g(x4,K4);
```

(%o10) $10000=d$

(%o11) $20000=d+4000\,c+16000000\,b+64000000000\,a$

(%o12) $50000=d+8000\,c+64000000\,b+512000000000\,a$

(%o13) $80000=d+10000\,c+100000000\,b+1000000000000\,a$

```
(%i14) l:algsys([g1,g2,g3,g4],[a,b,c,d]);
```

(%o14) $[[a=\frac{1}{16000000},b=-\frac{1}{8000},c=2,d=10000]]$

Kostenfunktion

```
(%i15) Kostenfunktion:g(x,K),l;
```

(%o15) $K=\frac{x^3}{16000000}-\frac{x^2}{8000}+2\,x+10000$

```
(%i16) K:rhs(Kostenfunktion);
```

(%o16) $\frac{x^3}{16000000}-\frac{x^2}{8000}+2\,x+10000$

```
(%i17) K(x):=''K;
```

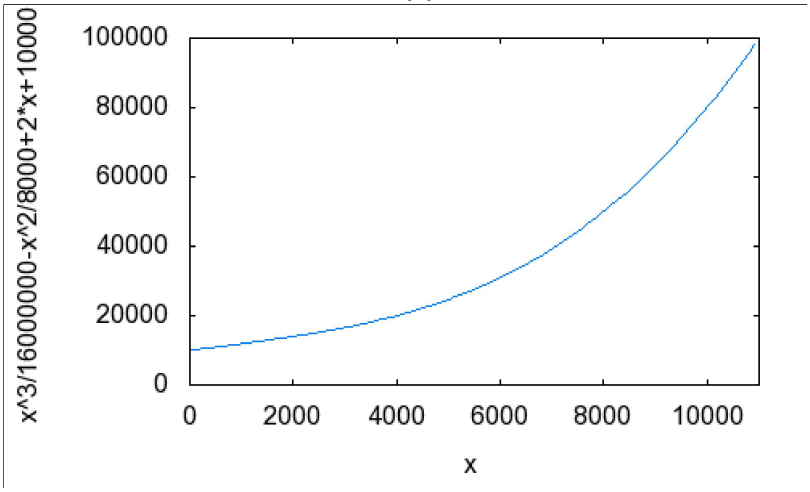
(%o17) $K(x):=\frac{x^3}{16000000}-\frac{x^2}{8000}+2\,x+10000$

Grafische Ausgabe

```
(%i18) wxplot2d([K(x)], [x,0,11000], [y,0,100000])$
```

plot2d: some values were clipped.

(%t18)



Durschnittskosten minimieren

```
(%i19) D(x):=K(x)/x;
```

(%o19) $D(x):=\frac{K(x)}{x}$

```
(%i19) ab:diff(D(x),x);
```

$$(\%o20)\ \frac{\frac{3\,x^2}{16000000}-\frac{x}{4000}+2}{x}-\frac{\frac{x^3}{16000000}-\frac{x^2}{8000}+2\,x+10000}{x^2}$$

```
(%i21) l:realroots(ab);
```

$$(\%o21)\ [x=\frac{156676063213}{33554432}]$$

Betriebsoptimum

```
(%i22) BO:x,l$
       BO:floor(BO+0.5);
(%o23) 4669
```

Langfristige Preisuntergrenze

```
(%i24) LPU:D(x),l$
       LPU:floor(LPU*100+0.5)/100.0;
(%o25) 4.92
```

```
(%i26) LPU:D(BO)$
       LPU:floor(LPU*100+0.5)/100.0;
(%o27) 4.92
```

Der Preis ist das 100-fache der langfristigen Preisuntergrenze

```
(%i28) p:100*LPU;
(%o28) 492.0
```

```
(%i29) U(x):=p*x;
```

$$(\%o29)\ U(x):=p\,x$$

Bestimmung der Gewinnzone

```
(%i30) g:U(x)=K(x);
```

$$(\%o30)\ 492.0\,x=\frac{x^3}{16000000}-\frac{x^2}{8000}+2\,x+10000$$

```
(%i31) l:realroots(g),numer;
(%o31) [x=-87559.73872962594,x=20.40805807709694,x=89539.33067151904]
```

```
(%i32) NS:x,l[2]$
       NS:floor(NS+0.5);
(%o33) 20
```

```
(%i34) NG:x,l[3]$
       NG:floor(NG+0.5);
(%o35) 89539
```

```
(%i36) Gewinnzone:[NS,NG];
(%o36) [20,89539]
```

Gewinnmaximierung

```
(%i37) G(x):=U(x)-K(x);
```

(%o37) $G(x):=U(x)-K(x)$

```
(%i38) ab:diff(G(x),x);
```

(%o38) $-\frac{3\,x^2}{16000000}+\frac{x}{4000}+490.0$

```
(%i39) l:realroots(ab);
```

(%o39) $[x=-\frac{1693104702727}{33554432},x=\frac{1737843945393}{33554432}]$

Die gewinnmaximale (Cournotsche) Menge

```
(%i41) xC:x,l[2]$
       xC:floor(xC+0.5);
```

(%o41) 51792

Der maximale Gewinn

```
(%i42) Gmax:G(xC);
```

(%o42) $1.7020416146431997\,10^{7}$