

Ableitungsregeln

☑ Dokumentnummer: DX1231

☑ 1 Ableitung der Potenzfunktion

☑ Figure 1:

Regel für die Ableitung der Potenzfunktion

(C15) $f(x) := x^2;$

(D15)

$$f(x) := x^2$$

(C16) $d: (f(x+h) - f(x)) / h;$

(D16)

$$\frac{(x+h)^2 - x^2}{h}$$

(C17) $\text{limit}(d, h, 0);$

(D17)

$$2x$$

☑ 1.1 Fall n = 2

☑ (%i5) $f(x) := x^2;$

☑ (%o5) $f(x) := x^2$

☑ (%i6) $d: (f(x+h) - f(x)) / h;$

☑ (%o6) $\frac{(x+h)^2 - x^2}{h}$

☑ (%i7) $\text{limit}(d, h, 0);$

☑ (%o7) $2x$

☑ 1.2 Fall n = 3

☑ (%i8) $f(x) := x^3;$

☑ (%o8) $f(x) := x^3$

☑ (%i9) $d: (f(x+h) - f(x)) / h;$

☑ (%o9) $\frac{(x+h)^3 - x^3}{h}$

(i10) `limit(d,h,0);`
 (o10) $3x^2$

1.3 Fall n = 4

(i11) `f(x):=x**4;`
 (o11) $f(x) := x^4$

(i12) `d:(f(x+h)-f(x))/h;`
 (o12) $\frac{(x+h)^4 - x^4}{h}$

(i13) `limit(d,h,0);`
 (o13) $4x^3$

1.4 Allgemeiner Fall

(i14) `f(x):=x**n;`
 (o14) $f(x) := x^n$

(i15) `d:(f(x+h)-f(x))/h;`
 (o15) $\frac{(x+h)^n - x^n}{h}$

(i16) `limit(d,h,0);`
 (o16) nx^{n-1}

2 Ableitung der Exponentialfunktion

2.1 Basis e

(i17) `f(x):=%e**x;`
 (o17) $f(x) := e^x$

(i18) `d:(f(x+h)-f(x))/h;`
 (o18) $\frac{e^{x+h} - e^x}{h}$

(i19) `limit(d,h,0);`
Is x an integer? no;
 (o19) e^x

□ 2.2 Basis allgemein

✓ (%i20) $f(x) := a^{**x}$;

[(%o20) $f(\mathbf{x}) := a^{\mathbf{x}}$

✓ (%i21) $d:(f(x+h)-f(x))/h$;

[(%o21) $\frac{a^{x+h} - a^x}{h}$

✓ (%i22) $\text{limit}(d,h,0)$;

[*Is x an integer?* no;

[(%o22) $a^x \log(a)$

□ 3 Trigonometrische Funktionen

□ 3.1 Sinusfunktion

✓ (%i23) $f(x) := \sin(x)$;

[(%o23) $f(\mathbf{x}) := \sin(\mathbf{x})$

✓ (%i24) $d:(f(x+h)-f(x))/h$;

[(%o24) $\frac{\sin(\mathbf{x} + \mathbf{h}) - \sin(\mathbf{x})}{h}$

✓ (%i25) $\text{limit}(d,h,0)$;

[(%o25) $\cos(\mathbf{x})$

□ 3.2 Cosinusfunktion

✓ (%i26) $f(x) := \cos(x)$;

[(%o26) $f(\mathbf{x}) := \cos(\mathbf{x})$

✓ (%i27) $d:(f(x+h)-f(x))/h$;

[(%o27) $\frac{\cos(\mathbf{x} + \mathbf{h}) - \cos(\mathbf{x})}{h}$

✓ (%i28) $\text{limit}(d,h,0)$;

[(%o28) $-\sin(\mathbf{x})$

□ 3.3 Tangensfunktion

$$\begin{aligned} & \text{(%i29) } f(x) := \tan(x); \\ & \text{(%o29) } f(\mathbf{x}) := \tan(\mathbf{x}) \end{aligned}$$

$$\begin{aligned} & \text{(%i30) } d:(f(x+h)-f(x))/h; \\ & \text{(%o30) } \frac{\tan(\mathbf{x} + \mathbf{h}) - \tan(\mathbf{x})}{\mathbf{h}} \end{aligned}$$

$$\begin{aligned} & \text{(%i31) } \text{limit}(d,h,0); \\ & \text{(%o31) } \frac{1}{\cos(\mathbf{x})^2} \end{aligned}$$

□ 4 Natürlicher Logarithmus

$$\begin{aligned} & \text{(%i32) } f(x) := \log(x); \\ & \text{(%o32) } f(\mathbf{x}) := \log(\mathbf{x}) \end{aligned}$$

$$\begin{aligned} & \text{(%i33) } d:(f(x+h)-f(x))/h; \\ & \text{(%o33) } \frac{\log(\mathbf{x} + \mathbf{h}) - \log(\mathbf{x})}{\mathbf{h}} \end{aligned}$$

$$\begin{aligned} & \text{(%i34) } \text{limit}(d,h,0); \\ & \text{(%o34) } \frac{1}{\mathbf{x}} \end{aligned}$$

□ 5 Summenregel

$$\begin{aligned} & \text{(%i35) } f(x) := u(x) + v(x); \\ & \text{(%o35) } f(\mathbf{x}) := u(\mathbf{x}) + v(\mathbf{x}) \end{aligned}$$

$$\begin{aligned} & \text{(%i38) } \text{diff}(f(x),x); \\ & \text{(%o38) } \frac{d}{d \mathbf{x}} v(\mathbf{x}) + \frac{d}{d \mathbf{x}} u(\mathbf{x}) \end{aligned}$$

□ 6 Produktregel

$$\begin{aligned} & \text{(%i39) } f(x) := u(x) * v(x); \\ & \text{(%o39) } f(\mathbf{x}) := u(\mathbf{x}) v(\mathbf{x}) \end{aligned}$$

$$\begin{aligned} & \text{(%i40) } \text{diff}(f(x),x); \\ & \text{(%o40) } u(\mathbf{x}) \left(\frac{d}{d \mathbf{x}} v(\mathbf{x}) \right) + v(\mathbf{x}) \left(\frac{d}{d \mathbf{x}} u(\mathbf{x}) \right) \end{aligned}$$

□ 7 Quotientenregel

```
(%i41) f(x):=u(x)/v(x);
```

```
(%o41) f(x):=
$$\frac{u(x)}{v(x)}$$

```

```
(%i42) diff(f(x),x),ratsimp;
```

```
(%o42) 
$$\frac{u(x)\left(\frac{d}{dx}v(x)\right)-v(x)\left(\frac{d}{dx}u(x)\right)}{v(x)^2}$$

```